

Having Forgotten To Doubt, "Modern Finance" Drove Us Insane

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Portfolio selection: Let's exhume the buried man! In his milestone paper "Portfolio Selection" published in the Journal of Finance in 1952, Harry Markowitz, the pioneer of "modern finance," recommends to use the Expected return-Variance (E-V) rule, both as a working hypothesis to explain investment behavior and as a guide to "investment" - as distinguished from speculative behavior. This rule implies that an actor who considers yield to be good, risk to be bad, and speculation to be banned, should diversify in such a way that his portfolio lies in the "efficient frontier." The idea is very simple. When building your portfolio, combine the securities in such a way that for a given expected return (E) of your portfolio, its variance (V) is minimum. This denies all the efficient combinations (E,V). Then, according to your degree of risk aversion, pick one such combination - a risk averse person will prefer a low E - low V portfolio, whereas a risk lover will choose a high E - high V combination. The beauty of this rule lies in its apparent readiness. Given a probability distribution of yields of the various securities, computing the set of efficient (E,V) combinations is straightforward. But this is misleading, because it dodges the main issue: Where do the expected returns and variances estimates come from? In other terms, how do we set our probability beliefs? At the end of the paper, Markowitz himself recognizes that he has been silent about the origin of these beliefs throughout:

"To use the E-V rule in the selection of securities, we must have procedures for ending reasonable expected returns and variances [...] I will not pursue the subject here, for this is "another story." It is a story of which I have read only the first page of the first chapter. In this paper we have considered the second stage in the process of selecting a portfolio. This stage starts with the relevant beliefs about the securities involved. We have not considered the first stage: the formation of the relevant beliefs on the basis of information."

Dodging the question of belief formation is murderous. It seems that fifty years from this paper, we are still stuck in "the first page of the first chapter." Quite ironically, after Markowitz's paper, such E-V rule has become ubiquitous in real-world finance, despite this inherent indeterminacy. Why does such belief indeterminacy matter? After all, is it so complicated to use a mix of statistical analysis and practical judgment, so that we derive sensible probability beliefs? In the first instance, one can use observed yields and volatilities from the past, to get statistical estimates of the true yields and variances. The problem is elsewhere. By emphasizing belief formation related to the expected returns and variances, we are missing the key point. Belief formation about world uncertainty is the issue, which is totally buried by Markowitz here. Markowitz implicitly assumes we live in a Gaussian world of "mild uncertainty," where price changes are characterized by stability around the average. In such a world, Markowitz's "E" and "V" are relevant objects. But what if randomness is not mild at all in our world? What if there is not such a thing as "value," and returns have infinite variance? In such a world, sensible people do not average but rather arbitrage between times and places. If so, by putting forward "E" and "V," maybe Markowitz leads us to wrongly interpret the world we live in.

Returns uncertainty from the empirical side: And there is good evidence that it is the case. If returns were Gaussian, we would have 68% of small price changes within one standard deviation of the mean, 95% within two standard deviations of the mean; and outliers (large changes) would be extremely rare: according to the Gaussian model, index swings of more than 7% should come once every 300,000 years... Rather, empirically, we observe too many large and too many small changes in the prices (this is what we call "fat tails"). Moreover, we observe irregular trends of large changes followed by clusters of small changes. That is, trouble runs in streaks (a wild day might be followed by a wilder day). This means that "persistence" is far larger than expected, would the world be Gaussian. As such - at least for investing horizons from two hours to six months, the Gaussian hypothesis is the wrong interpretation.

Uncertainty in the finance world: Where do we stand? What is the correct model of uncertainty in finance then? There is no definite answer to that paramount question. Actually, two distinct routes are possible.

Nonstationarity: The first one assumes nonstationarity. For instance, the widespread use of GARCH models is to capture the foregoing phenomenon of persistence, while staying within the Gaussian boundaries. The idea is to introduce changes in volatility - that is, instead of considering one single Gaussian distribution for the returns, consider multiple ones, each characterized by its own level of volatility. When real world volatility soars (resp settles), make the Gaussian curve grow (resp shrink). This indeed enables to fit the data. Poissonian uncertainty has also been suggested as a model to replace the Gaussian (Brownian) model. Gaussian risk involves a high probability of a small change, while Poissonian risk involves a small probability of a large change (jump).

The multifractal model: The second route is the one taken by Benoît Mandelbrot, the father of fractal geometry. It avoids to assume nonstationarity and rather, he proposes real world randomness to be best described as "slow." The Gaussian model entails a randomness that is too mild. Conversely, "wild randomness" is characterized by an extreme degree of unpredictability: tails are huge, "everything can happen," whereby there is no way to forecast the returns (both expectation and variance of price changes are infinite). Slow randomness is in between: there is no stability around the average, and tails are fatter than in the Gaussian world. Mandelbrot's model of real world uncertainty is remarkably elegant - and I'm not (only) saying that because Mandelbrot is French. Not only his framework generates the fat tails and persistence phenomena observed in real data, it also suggests using quantitative tools to rigorously measure (1) how fat the tails are, and (2) the degree of persistence in the returns.

α and H : Two parameters summarize these two dimensions, α for the size of the returns, and H for their sequences (dependence). The first parameter comes from modeling the tails with a Power Law. For x large (we are at the tails), it sets the ratio of probability of a return larger than nx over the probability of a return larger than x to be $n^{-\alpha}$. Intuitively, the smaller α , the fatter the tails (i.e., the larger the instability: the realization of an outlier moves the average). With a Gaussian distribution, α equals 2; under wild randomness, it is 1. α in between points to a world of slow randomness, the one we presumably live in many instances. The second parameter is very intuitive too. It says something about the sequences of the price changes (runs) rather than their size. The question is how much the past shapes. In a Gaussian world, over a given period (say, 10 years), the range between the highest return and the lowest one is $\sqrt{5}$ times the empirical standard deviation of the returns from one year to the next. However, when randomness is characterized by persistence, the high-to-low range widens not by a square-root law but as a H power, with H larger than 1/2. This means long term dependence, and captures the fact that turbulence clusters.

Blueprint: Assessing real uncertainty is like constructing "dikes" No model is universal. Plainly, interpreting properly the nature of the randomness we face in our investment decision is context-specific. So, how do we choose between the models? We don't have to be blind about uncertainty. Once we no longer take for granted the Gaussian assumption, all we have to do is appraising properly the world we invest in. That is, we can track - and perhaps forecast - how turbulent the market is becoming, using fractal geometry and the α and H measures. In "The Prince", N. Machiavelli compares "Fortune" to a violent river, and suggests our constructing dikes to protect ourselves. What does it mean for finance to construct such dikes? Just that we need to assess α and H .

And in practice? If α is smaller than 2, then we should definitely think again about the E-V rule. (Same thing if H is different from 1/2.) If so, we should ignore "E" and "V" and merely focus on both α and H , with which to evaluate true risk. Sadly, things are not that simple, because measuring α and H is in practice very difficult. In 1991 Andrew Lo reported that Mandelbrot's tests for H can confound long-term memory with the effects of short-term memory. And the measures of H are not robust: there is no consensus as for the S&P 500 index for example: the estimates for H vary from 0:53 to 0:74.

Furthermore, the degree of dependence varies a lot from one type of financial asset to another (gold prices, oil markets, foreign exchanges, might have long memory, whereas cotton, British government bonds, do not). So, overall, it is unrealistic to hail α and H as new yardsticks for finance.

We'd better acknowledge our ignorance: Doubt is good To be honest, it is impossible to claim with certainty that one model is the correct one at a specific moment in time. Accounting for model uncertainty is the hallmark of modern econometrics and (truly) modern Finance. Is it a retreat into - at least lucid-blindness? I don't think so. The only relevant question is referred to by Nassim Taleb as "tinkering:" How to make sound investment choices in a world we don't understand? Modern finance tackles this problem, by putting forward rules of behavior under "ambiguity." By "ambiguity," we academics precisely refer to these situations of missing information about "the rules" of financial investment, the one we model as a big game of betting under unknown odds. F. Nietzsche wrote in his "Ecce Homo:" "No doubt, certainty is what drives one insane." I think "modern finance" drove us insane because it has forgotten to doubt.